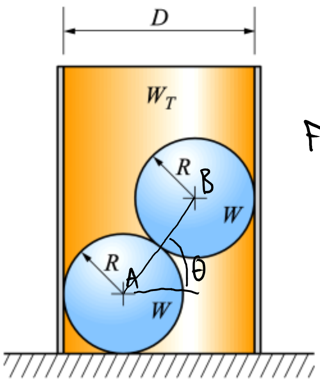
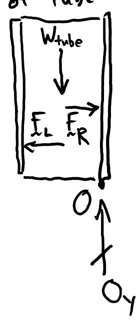


Two marbles, ~~which~~ [each] of radius R and weight ~~W~~ [W_M], are placed inside a hollow thin-walled tube of diameter D. Note that $D < 4R$, so that only one marble touches the floor. Find the minimum weight ~~W_T~~ [W_{tube}] of the tube such that it will not turn over. All surfaces are smooth.

FBD of tube



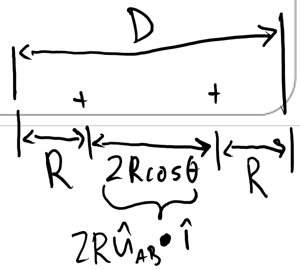
Reactions at point O, at the instance when it begins to tip over.

Note: No horizontal reaction at O.

Step 1. Find θ. See that $|\underset{\sim}{r}_{AB}| = 2R$

$$\underset{\sim}{r}_{AB} = 2R \cdot \hat{u}_{AB} = 2R \cdot (\cos\theta\, \hat{i} + \sin\theta\, \hat{j})$$

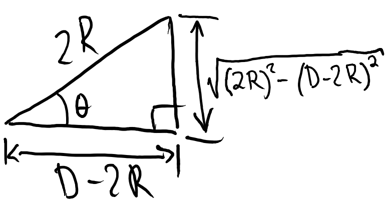
Geometry. Relate R & D



$$\Rightarrow 2R + 2R\cos\theta = D$$

$$2R(1+\cos\theta) = D \quad \Rightarrow \boxed{\cos\theta = \frac{D}{2R} - 1 = \frac{D-2R}{2R}}$$

solved for cos θ.



We might need sin θ & tan θ.

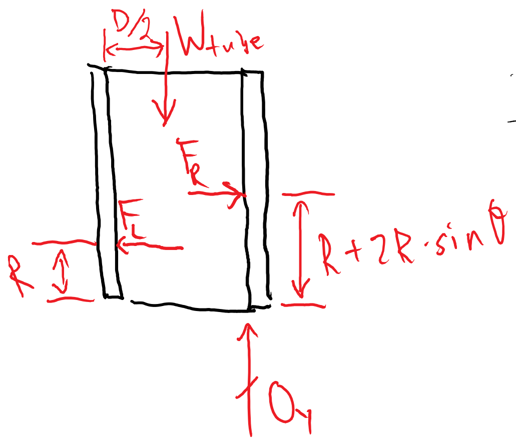
$$\cos\theta = \frac{adj}{hyp.} = \frac{D-2R}{2R}$$

$$\sin\theta = \frac{\sqrt{4RD - D^2}}{2R}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{4RD-D^2}}{D-2R}$$

FBD of tube

Apply equilibrium eqns:



Apply equilibrium eqns:

$$\Sigma F_x = F_R - F_L = 0$$

$$F_R = F_L = F_{wall}$$

$$\Sigma F_y = O_y - W_{tube} = 0$$

$$O_y = W_{tube}$$

$$\Sigma M_O = -(R + 2R\sin\theta)F_{wall} + R \cdot F_{wall} + W_{tube}\cdot\frac{D}{2} = 0$$

$$-2R\cdot F_{wall}\cdot\sin\theta + W_{tube}\cdot\frac{D}{2} = 0$$

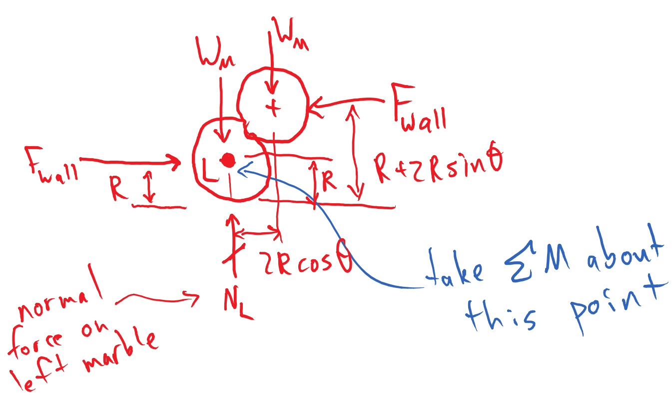
$$\sin\theta = \frac{\sqrt{4RD - D^2}}{2R}$$

$$\boxed{\frac{D\cdot W_{tube}}{2} = F_{wall}\sqrt{4RD - D^2}}$$

At the instance where tipping begins

Now, relate F_{wall} to W_m ← weight of the marble

Treat the marbles as one rigid body



$$\Sigma M_L = -(2R\cos\theta)\cdot W_m + (2R\cdot\sin\theta)\cdot F_{wall} = 0$$

$$\boxed{F_{wall} = \frac{2R\cdot\cos\theta\cdot W_m}{2R\cdot\sin\theta} = W_m\cdot\frac{\cos\theta}{\sin\theta} = \frac{W_m}{\tan\theta}}$$

$$\frac{\cancel{2}R\cdot\sin\theta}{} = W_m \cdot \frac{}{\sin\theta}\left(\frac{}{\tan\theta}\right)$$

$$\frac{D\cdot W_{tube}}{2} = F_{wall}\sqrt{4RD-D^2}$$

$$= \frac{W_m}{\tan\theta}\cdot\sqrt{4RD-D^2}$$

$$= \frac{W_m(D-2R)}{\cancel{\sqrt{4RD-D^2}}}\cancel{\sqrt{4RD-D^2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{4RD-D^2}}{D-2R}$$

$$\tan\theta = \frac{\sqrt{4RD-D^2}}{D-2R}$$

$$\boxed{W_{tube} = \frac{2}{D}\cdot(D-2R)\cdot W_m}$$

force — dim'less — force

Equilibrium in three-dimensional bodies

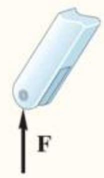
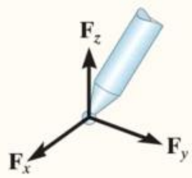
TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable. Direction is known, magnitude is not.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Page 22

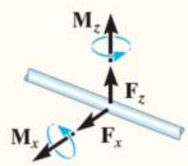
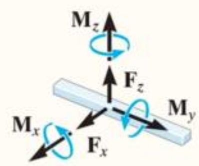
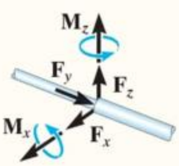
Monday, February 13, 2017 12:55 AM

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(3)  roller (assume smooth)		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components. No rotational constraint ⇒ no reaction moments

Monday, February 13, 2017 12:55 AM

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.
(6)  single journal bearing with square shaft		Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(7)  single thrust bearing		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

Page 24

Monday, February 13, 2017 12:55 AM

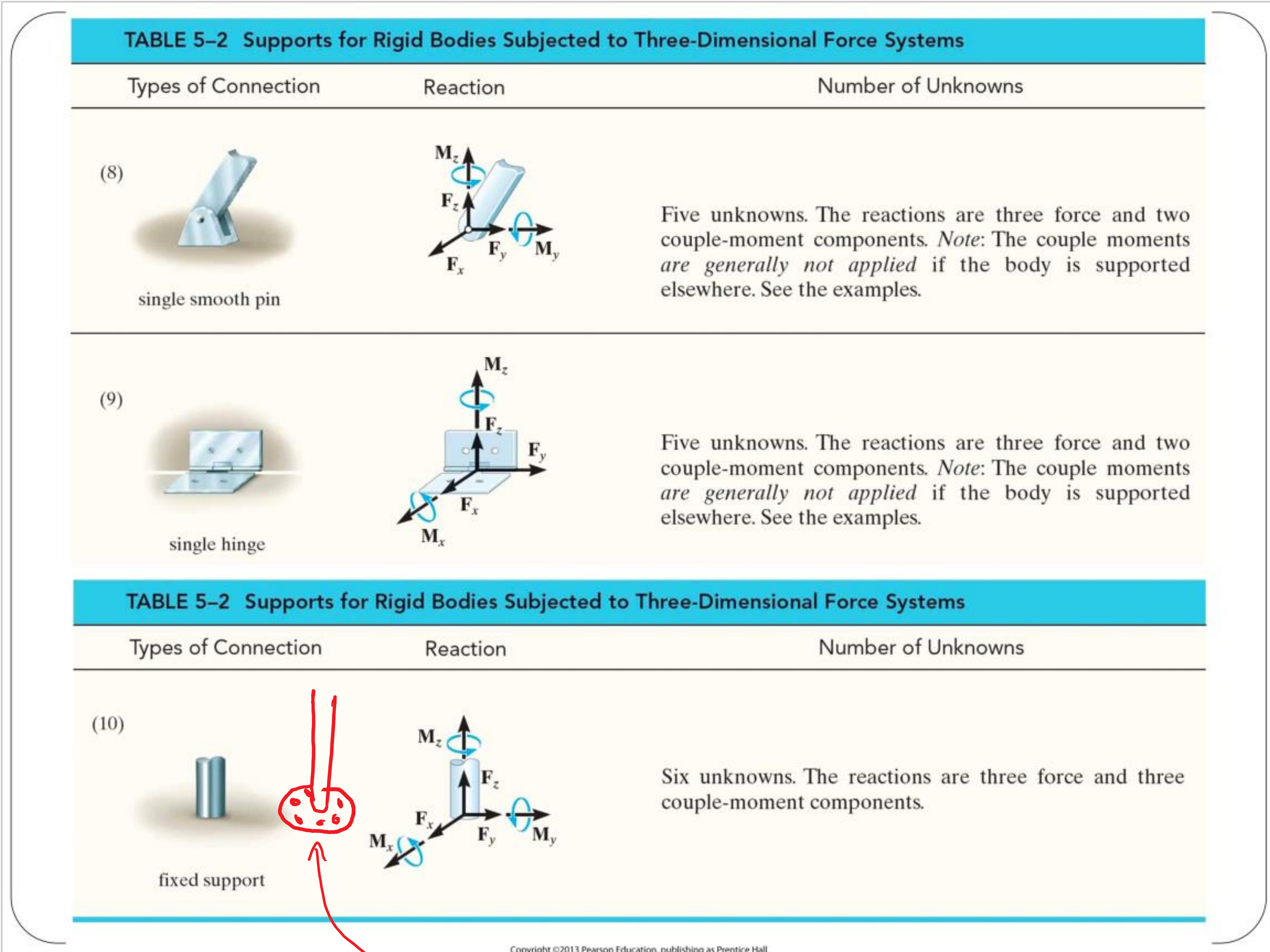


TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(8) single smooth pin	$\mathbf{M}_z$, $\mathbf{F}_z$, $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{M}_y$	Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(9) single hinge	$\mathbf{M}_z$, $\mathbf{F}_z$, $\mathbf{F}_y$, $\mathbf{F}_x$, $\mathbf{M}_x$	Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

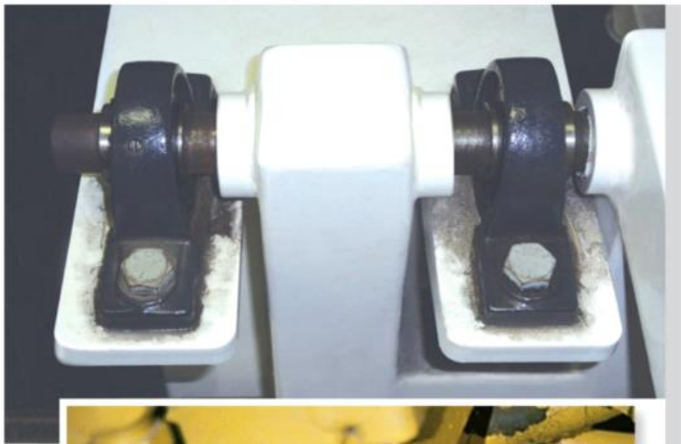
Types of Connection	Reaction	Number of Unknowns
(10) fixed support	$\mathbf{M}_z$, $\mathbf{F}_z$, $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{M}_y$, $\mathbf{M}_x$	Six unknowns. The reactions are three force and three couple-moment components.

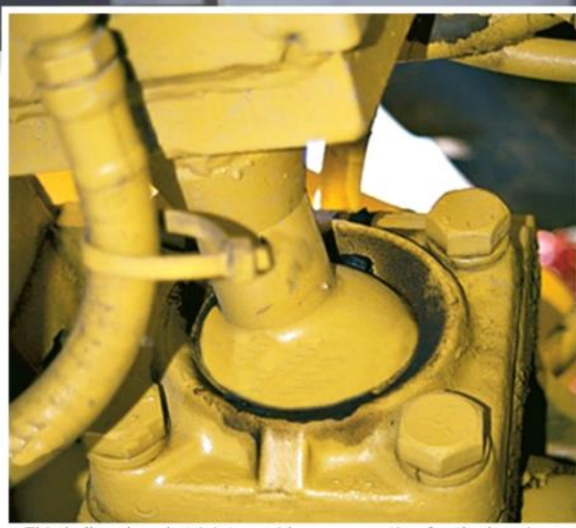
typically, a bolted flange is used – even though the diagram does not show one

Page 25

Monday, February 13, 2017 12:55 AM

The journal bearings support the ends of the shaft. (5)

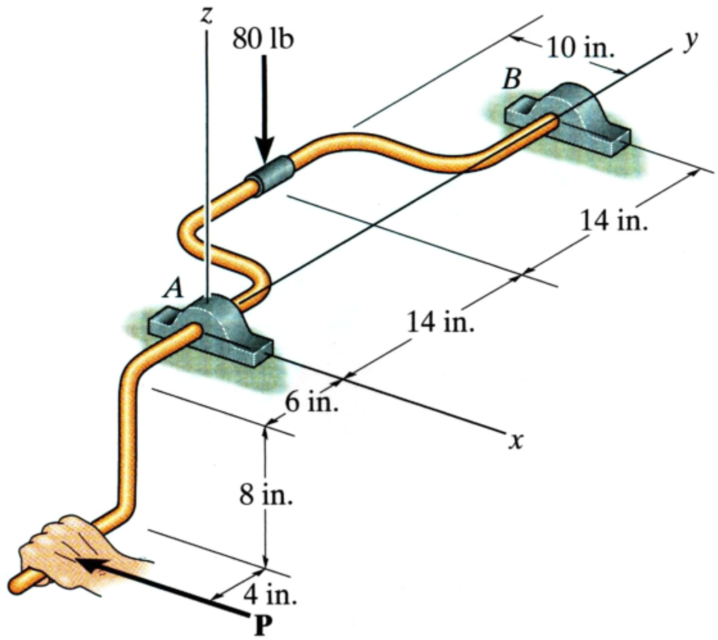




This ball-and-socket joint provides a connection for the housing of an earth grader to its frame. (4)

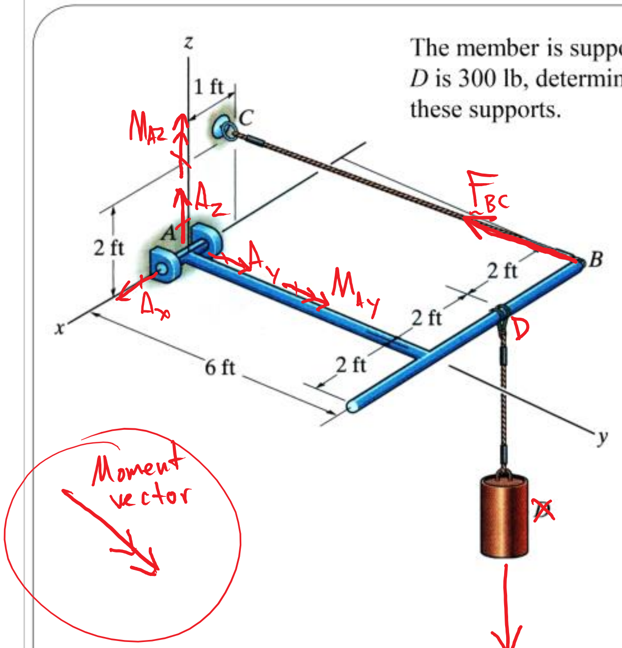
This pin is used to support the end of the strut used on a tractor. (8)

This thrust bearing is used to support the drive shaft on a machine. (7)



A vertical force of 80 lb acts on the crankshaft. Determine the horizontal equilibrium force *P* that must be applied to the handle and the *x*, *y*, *z* components of force at the smooth journal bearing *A* and the thrust bearing *B*. The bearings are properly aligned and exert only force reactions on the shaft.

Monday, February 13, 2017 12:55 AM



The member is supported by a pin at *A* and cable *BC*. If the load at *D* is 300 lb, determine the *x*, *y*, *z* components of the reactions at these supports.

$\underset{\sim}{W}_D = -300\hat{k}$ lbs

Reactions

A_x, A_y, A_z

M_{Ay}, M_{Az}

1 add'l unknown

$\underset{\sim}{F}_{BC} = F_{BC} \cdot \hat{u}_{BC}$

← magnitude is unknown

Position vectors

$\underset{\sim}{r}_{AB} = (-4\hat{i} + 6\hat{j})$ ft

$\underset{\sim}{r}_{AD} = (-2\hat{i} + 6\hat{j})$ ft

$\underset{\sim}{r}_{BC} = (3\hat{i} - 6\hat{j} + 2\hat{k})$ ft

$$\hat{u}_{BC} = \frac{(3\hat{i} - 6\hat{j} + 2\hat{k})}{\sqrt{9 + 36 + 4}}$$

(9 + 36 + 4 = 49)

$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

Force vectors

$\underset{\sim}{F}_{BC} = F_{BC}\left(\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$

$\underset{\sim}{W}_D = -300\hat{k}$ lbs

$\underset{\sim}{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$